

On the D-brane solutions in Gödel Universe

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Abstract

We present a class of supersymmetric Gödel solutions in string theory from the non-standard intersection of branes in supergravities. Such solutions are obtained by applying a T-duality on the known solutions in PP-wave spacetime. We further present classical solutions of supersymmetric D-brane in Gödel universes arising from the PP-wave in the near horizon geometry of stack of D5-branes and from the new isometries of H_6 PP-wave background. These branes are supported by multiple constant Neveu-Schwarz and Ramond-Ramond field strengths.

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1 Introduction and summary

Gödel universe is a homogeneous rotating cosmological solution of Einstein's equations with pressureless matter and negative cosmological constant, which played an important role in the conceptual development of general theory relativity. In [1] an M-theory solution of Gödel universe type has been found out and it was shown to preserve 20 supersymmetries. Furthermore it generates Ramond-Ramond (RR) fluxes when compactified down to 10-dimensional type IIA string theory. They contain some unphysical features like the closed time like curves (CTC), but the problem was resolved geometrically in [2] in the context of spinning deformation of (D1-D5) system. Further in [3], it was argued that the principle of holography remedy this problem and protect the chronology in the Gödel universe background. It was further shown that they are related to PP-wave background by a T -duality transformation. In this connection in [4] a large class of solutions were found out in the context of string theory in PP-wave background and corresponding properties of such spacetime including that of supersymmetries has been analyzed in details both in 10 and 11 dimensions. These class of solutions were obtained by applying T and S -duality transformations in the relevant solutions in PP-wave background. The string theory spectrum has also been studied by invoking the idea of quantization of PP-wave in light cone gauge. Further it was also noticed that the supergravity solutions of D5-branes in a type IIB PP-wave background[5] (coming from $AdS_3 \times S^3$ geometries) after a T-duality transformation can give new localized D4-brane solutions in the Gödel universe. They were also obtained by looking at the relevant boundary conditions in the open string constructions in PP-wave[6] and then applying T-dualities. This generates the mixed boundary conditions. Also in [7], using the duality described in [3], the string quantization has been studied in the ten dimensional description of these solutions and yet another mechanism has been proposed to resolve the CTCs within string theory. In this paper we would like to obtain new supersymmetric Gödel backgrounds by applying T-dualities in a class of PP-wave background which was derived from the Penrose limit on the non-standard intersection of D-branes in supergravities. Indeed in [8] a large class of PP-wave backgrounds were obtained from the non-standard brane intersections whose near horizon geometry was essentially Anti-de Sitter (AdS). The resulting PP-waves are shown to be supported by multiple constant RR and NS-NS field strengths and they are interesting in their own right. These solutions are different from the known near horizon and PP-wave limit of the usual intersecting (like D1-D5) branes in supergravities. The main difference stems from the fact that in this case the Harmonic functions of branes depend on the relative transverse space. We have found out the new Gödel universe backgrounds by applying T-dualities on these class of PP-wave solutions. We take two examples, the (D1/D5/D5) system and the D3/D5/D5/ND5/NS5 intersecting brane system and have obtained new Gödel universe backgrounds. These solutions are different from the known solutions as they contain multiple RR and NS-NS fields, but keeps the "Gödel structure" is still intact.

Next, we find out some new supergravity solutions of D-branes in type IIA string theory from the known solutions of D-branes in PP-wave backgrounds¹. First we have taken the example of D5-branes in the near horizon and PP-wave background of a stack of D5-branes in type IIB string theory. This solution was found in [11] and was shown to be 1/4 supersymmetric. We apply T-duality and present a D4-brane solution in Gödel universe (the so-called $n = 1$ Gödel model) and examine the fate of unbroken supersymmetry by solving the gravitino and dilatino variations explicitly. Further we have uplifted this solution to M5-brane in M-theory. Our next example is a D2-brane in Gödel universe which is obtained from a D3-brane in PP-wave in the presence of both RR and NS-NS 3 form field strengths found in [10]. The rest of the paper is organized as follows. In section-2, we find out new supersymmetric Gödel solutions from intersecting branes whose near horizon geometry are of AdS type. In section-3, we find out new supergravity solutions for D-branes in Gödel universes from the corresponding branes in PP-wave backgrounds. Section-4 is devoted to the analysis of unbroken spacetime supersymmetry. Finally in section-5, we conclude with some remarks.

2 Gödel Universes from intersecting branes

In this section we will find out new Gödel models from the PP-wave background of non-standard brane intersections in supergravities. The AdS structure in the near horizon geometry of such intersections arises from the fact the Harmonic function for each participating brane depends on the relative transverse space rather than the overall transverse space. The first example we consider is the intersecting (D1/D5/D5)-brane system that couple to three form R-R field strengths in D=10. The relevant metric and other fields are given by [16]

$$\begin{aligned}
ds_{10}^2 &= G^{-3/4}(F\tilde{F})^{-1/4} \left(-dt^2 + dx^2 + GFdy_i^2 + G\tilde{F}d\tilde{y}_i^2 \right) \\
F_{(3)} &= e^\phi * (\tilde{F}dt \wedge dx \wedge d^4\tilde{y} \wedge dF^{-1}) + e^\phi * (Fdt \wedge dx \wedge d^4y \wedge d\tilde{F}^{-1}) \\
&+ dt \wedge dx \wedge dG^{-1}, \quad e^{2\phi} = \frac{F\tilde{F}}{G}, \quad G = F\tilde{F},
\end{aligned} \tag{2.1}$$

where y_i and \tilde{y}_i are the coordinates in the relative transverse space of the stack of D5-branes, $F = 1 + \frac{Q}{y^2}$, $\tilde{F} = 1 + \frac{Q}{\tilde{y}^2}$ are the harmonic functions of the branes. This particular solution has the property that the dilaton vanishes. The near horizon geometry of such a solution is $AdS_3 \times S^3 \times S^3 \times S^1$. The PP-wave geometry was obtained in [8], after taking a suitable Penrose limit on (2.1) and it is given by

$$ds_{10}^2 = -2dx^+dx^- + H(dx^+)^2 + \sum_{i=1}^8 dx_i^2, \quad F_3^{(RR)} = dx^+ \wedge \Phi_{(2)},$$

¹D-brane supergravity solutions in NS-NS and R-R PP-waves have been discussed in, for example in [5][9][10][11][12] [13][14][15]

$$\begin{aligned}
H &= -\mu^2(x_1^2 + x_2^2) - \frac{\mu^2}{2} \cos^2 \alpha (x_3^2 + x_4^2) - \frac{\mu^2}{2} \sin^2 \alpha (x_5^2 + x_6^2) , \\
\Phi_{(2)} &= 2\mu dx_1 \wedge dx_2 + \sqrt{2}\mu \cos \alpha \, dx_3 \wedge dx_4 - \sqrt{2}\mu \sin \alpha \, dx_5 \wedge dx_6 , \quad (2.2)
\end{aligned}$$

where α is the angle of rotation between the coordinates of two spheres in the transverse space of branes. We wish to find out the Gödel spacetime of this geometry. We shall follow the same procedure of getting a Gödel from PP-wave by applying a T-duality as described in [4]. The first step is to do the following coordinate transformation

$$x^+ = x^0 + x^9, \quad x^- = \frac{x^0 - x^9}{2}, \quad x^{2k-1} + ix^{2k} \rightarrow (x^{2k-1} + ix^{2k})e^{-i\mu_k x^+}, \quad k = 1, 2, 3. \quad (2.3)$$

With the above transformation, the new metric looks like

$$ds^2 = -(dx^0)^2 + (dx^9)^2 + \sum_{i=1}^8 dx_i^2 - 2 \sum_{i,j=1}^6 J_{ij} x_i dx_j (dx^0 + dx^9) , \quad (2.4)$$

where

$$J_{12} = \mu = -J_{21} , \quad J_{34} = \frac{\mu}{\sqrt{2}} \cos \alpha = -J_{43} , \quad J_{56} = \frac{\mu}{\sqrt{2}} \sin \alpha = -J_{65} . \quad (2.5)$$

The next step is to apply a T -duality along the x^9 direction ². The new metric and fields after the T -duality become

$$\begin{aligned}
ds^2 &= -(dx^0 + \sum_{i,j=1}^6 J_{ij} x_i dx_j)^2 + (dx^9)^2 + \sum_{i=1}^9 dx_i^2 , \quad H_{129} = -F_{0129} = F_{12} = -2\mu , \\
H_{349} &= -F_{0349} = F_{34} = -\sqrt{2}\mu \cos \alpha , \quad F_{0569} = H_{569} = -F_{56} = -\sqrt{2}\mu \sin \alpha . \quad (2.6)
\end{aligned}$$

This background is different from the known examples of [4]. It is also important to note that this background preserves 1/2 supersymmetry only for $\alpha = \pi/4$ which is expected from [8]. Our next example is a non-standard intersection of D3/D5/D5/NS5/NS5 branes. This near horizon geometry was found out to be $AdS_3 \times S^2 \times S^2 \times T^3$. The Penrose limit was taken and the resulting pp-wave background has been written in [8]. We wish to write it once for our future reference

$$\begin{aligned}
ds^2 &= -2dx^+ dx^- - \mu^2 (x_2^2 + x_3^2 + 2 \cos^2 \alpha x_3^2 + 2 \sin^2 \alpha x_4^2) (dx^+)^2 + dx_i^2 , \\
F_{+1268} &= 2\mu , \quad F_{+36} = H_{+38} = \sqrt{2}\mu \cos \alpha , \quad F_{+48} = H_{+46} = -\sqrt{2}\mu \sin \alpha . \quad (2.7)
\end{aligned}$$

For our purpose we will set $\alpha = \pi/4$. With this choice, the metric and other fields can be read off as

$$\begin{aligned}
ds^2 &= -2dx^+ dx^- - \mu^2 \sum_{i=1}^4 x_i^2 (dx^+)^2 + \sum_{i=1}^8 dx_i^2 , \\
F_{+1268} &= 2\mu , \quad F_{+36} = H_{+38} = \mu , \quad F_{+48} = H_{+46} = -\mu . \quad (2.8)
\end{aligned}$$

²The T-duality transformation can be found out for example in [17]

After applying T -duality along x^9 direction as described earlier, we end of with the following form of the metric and other resultant field strengths as

$$\begin{aligned}
ds^2 &= -(dx^0 + \sum_{i,j=1}^4 J_{ij} x_i dx_j)^2 + (dx^9)^2 + \sum_{i=1}^8 dx_i^2, \quad H_{129} = H_{349} = -2\mu, \\
F_{0369} &= \mu = -F_{0489}, \quad F_{1268} = F_{3457} = 2\mu, \quad F_{36} = -\mu = -F_{48}, \\
H_{038} &= \mu = H_{938}, \quad H_{046} = -\mu = H_{946}
\end{aligned} \tag{2.9}$$

3 D-brane solution in Gödel Universes

In this section, we would like to write down the D4-brane solutions in the Gödel universe of $n = 2$ type presented in [4], which will be used in the next section to study the supersymmetry. The metric, dilaton and various field strengths of a stack of D4-branes is given by [4]

$$\begin{aligned}
ds^2 &= f_4^{-1/2} \left(-(dt + \mu \sum_{i=1}^4 J_{ij} x^i dx^j)^2 + \sum_{i=1}^4 (dx^i)^2 \right) + f_4^{1/2} \sum_{m=5}^9 (dx^m)^2 \\
e^{2\phi} &= f_4^{-1/2}, \quad F_{12} = F_{34} = -2\mu, \quad H_{129} = H_{349} = -2\mu, \quad F_{0129} = F_{0349} = 2\mu, \\
F_{mnpq} &= \epsilon_{mnpqr} \partial_r f_4, \quad f_4 = 1 + \frac{N g_s l_s^3}{r^3}, \quad r^2 = \sum_{m=5}^9 (x^m)^2,
\end{aligned} \tag{3.1}$$

where $J_{12} = -J_{21} = J_{34} = -J_{43} = 1$ and f_4 is the harmonic function of the D4-brane in the transverse five-space. One can observe that the presence of various field strengths symmetrically along the x^1, x^2 and correspondingly along the x^3, x^4 directions. We will see that this structure plays a crucial role in the supersymmetry analysis of the D4-brane solution. Now we would like to present further examples of D-brane solution in the Gödel universe models. Our first example is a D4-brane in the so-called $n = 1$ Gödel universe model. This is obtained by applying T -duality along isometry directions of the D5-brane in a PP-wave background that arises from the near horizon and Penrose limit of a stack of coincident D5-branes and is dual to the PP-wave background of Nappi-Witten model. The supergravity solution of D-branes were presented in [11]. In particular the 1/4 supersymmetric D5-brane is written as [11]

$$\begin{aligned}
ds^2 &= f_5^{-1/2} \left(-2dx^+ dx^- - \mu^2 \sum_{i=1}^2 x_i^2 (dx^+)^2 + \sum_{a=1}^4 (dx^a)^2 \right) + f_5^{1/2} \sum_{m=5}^8 (dx^m)^2, \\
e^{2\phi} &= f_5^{-1}, \quad F_{+12} = 2\mu, \quad F_{mnp} = \epsilon_{mnpq} \partial_q f_5, \quad f_5 = 1 + \frac{N g_s l_2^2}{r^2}, \quad r = \sqrt{\sum_{m=5}^8 (x^m)^2}.
\end{aligned} \tag{3.2}$$

The spacetime supersymmetry was analyzed by solving the dilatino and gravitino variations explicitly and it was found out that in addition to the flat space D5-brane supersymmetry condition if a ‘necessary’ condition $\Gamma^{\hat{t}}\epsilon = 0$ acts on the killing spinors, then all variations are satisfied giving a solution for the spinors which preserves eight supercharges. Applying a T -duality along x^9 as described in the last section, we get the following form of the metric, field strengths and dilaton for the ‘localized’ D4-brane in Gödel model.

$$\begin{aligned}
ds^2 &= f_4^{-1/2} \left(-(dt + \mu \sum_{i=1}^2 J_{ij} x^i dx^j)^2 + \sum_{a=1}^4 (dx^a)^2 \right) + f_4^{1/2} \sum_{m=5}^9 (dx^m)^2 \\
e^{2\phi} &= f_4^{-1/2}, \quad F_{12} = -2\mu, \quad H_{129} = -2\mu, \quad F_{0129} = 2\mu, \quad F_{mnpq} = \epsilon_{mnpqr} \partial_r f_4, \\
J_{12} &= 1 = -J_{21}, \quad f_4 = 1 + \frac{N g_s l_s^3}{r^3}, \quad r^2 = \sum_{m=5}^9 (x^m)^2.
\end{aligned} \tag{3.3}$$

We have checked that the solution presented above solves all type-IIA field equations. Next we would like to get a M5-brane solution starting from the D4-brane solution presented above in the Gödel model. Using the well known relation between the 10d and 11d metric:

$$ds_{11}^2 = e^{-\frac{2\Phi}{3}} ds_{10}^2 + e^{\frac{4\Phi}{3}} (dx_{11} + A_\mu dx^\mu)^2, \tag{3.4}$$

where ds_{11}^2 and ds_{10}^2 represent the metric in eleven and ten dimensions respectively, and A_μ is the one-form field (which is zero in the present case). One can easily see that the M5-brane solution is given by

$$\begin{aligned}
ds^2 &= f^{-1/3} \left(-2dx^+ dx^- - \mu^2 \sum_{i=1}^2 (x^i)^2 (dx^+)^2 + \sum_{a=1}^4 (dx^a)^2 \right) + f^{2/3} \sum_{m=5}^9 (dx^m)^2, \\
F_{+129} &= 2\mu, \quad F_{mnpq} = \epsilon_{mnpqr} \partial_r f, \quad f = 1 + \frac{N l_p^3}{r^3},
\end{aligned} \tag{3.5}$$

with l_p being the eleven dimensional Plank length. In writing down the above solution in the x^+, x^- -coordinates, we have made the following change of variables

$$x^1 + ix^2 \rightarrow (x^1 + ix^2) e^{-2\mu x^+}. \tag{3.6}$$

The solution can directly be obtained from the PP-wave solution by uplifting it to eleven dimensions. Note that in absence of any D-brane charges, if we apply T dualities along x^3 and x^4 directions, we get the following form of metric and RR fields

$$\begin{aligned}
ds^2 &= -2dx^+ dx^- - \mu^2 \sum_{i=1}^2 (x^i)^2 (dx^+)^2 + \sum_{m=1}^8 (dx^m)^2, \\
F_{+1234} &= F_{+5678} = 2\mu.
\end{aligned} \tag{3.7}$$

Once again by applying a T -duality along the x^9 direction as before we get the following Godel metric and other field strengths

$$\begin{aligned} ds^2 &= - \left[dt + \mu(x^1 dx^2 - x^2 dx^1) \right]^2 + \sum_{m=1}^9 (dx^m)^2 \\ F_{1234} &= F_{5678} = 2\mu, \quad H_{129} = 2\mu, \end{aligned} \quad (3.8)$$

Next we would like to find a D2-brane solution in a Gödel model. The D2-brane can be obtained by applying a T -duality along a localized D3-brane solution described in [10]. Note that this D3-brane solution was obtained by applying successive T -dualities along the new isometry directions of the localized D5-brane solution of [10] by following [18]. In stead of going into the details of construction we present here the final form of D3-brane solution in the presence of various R-R and NS-NS fluxes as

$$\begin{aligned} ds^2 &= f_3^{-1/2} \left(-2dx^+ dx^- - 4\mu^2 [x_1^2 + x_2^2] (dx^+)^2 + dx_1^2 + dx_2^2 \right) + f_3^{1/2} \left(dr^2 + r^2 d\Omega_5^2 \right), \\ F_{+31} &= F_{+42} = 2\mu, \quad H_{+41} = H_{+32} = 4\mu, \quad F_{mnpqr} = \epsilon_{mnpqrs} \partial_s f_3, \quad f_3 = 1 + \frac{Ng_s l_s^4}{r^4}, \end{aligned} \quad (3.9)$$

where f_3 is harmonic function in the transverse six space. By applying T -duality along x^9 -direction as before, we get the following metric and other field strengths

$$\begin{aligned} ds^2 &= f_2^{-1/2} \left[- \left(dx^0 + 2\mu \sum_{i,j=1}^2 J_{ij} x^i dx^j \right)^2 + \sum_{i=1}^2 (dx^i)^2 \right] + f_2^{1/2} \sum_{m=3}^9 (dx^m)^2, \\ e^{2\phi} &= f_2^{1/2}, \quad A_{012} = f_2^{-1}, \quad F_{0329} = F_{0429} = 2\mu, \quad F_{31} = F_{42} = -2\mu, \\ H_{041} &= H_{941} = 4\mu = -H_{129}, \quad H_{032} = H_{932} = 4\mu, \quad f_2 = 1 + \frac{Ng_s l_s^5}{r^5}, \quad r^2 = \sum_{m=3}^9 (x^m)^2. \end{aligned} \quad (3.10)$$

Once again we have checked that the localized D2-brane solution above solves all type-IIA field equations of motion. Other D-branes and their bound states can be found out by applying T -dualities along various isometries of the solution presented here.

4 Spacetime Supersymmetry Analysis

In this section, we will analyze the fate of the unbroken spacetime supersymmetry of the D-brane solutions presented above by solving the dilatino and gravitino variations explicitly. The supersymmetry variation of the dilatino and gravitino fields in

type IIA supergravity in string frame is given by [19, 20].

$$\delta\lambda = \frac{1}{2}(\Gamma^\mu\partial_\mu\Phi - \frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho})\epsilon + \frac{1}{8}e^\Phi(5F^{(0)} - \frac{3}{2!}\Gamma^{\mu\nu}F_{\mu\nu}^{(2)} + \frac{1}{4!}\Gamma^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma})\epsilon, \quad (4.1)$$

$$\delta\Psi_\mu = \left[\partial_\mu + \frac{1}{8}(w_{\hat{a}\hat{b}} - H_{\hat{a}\hat{b}})\Gamma^{\hat{a}\hat{b}}\right]\epsilon + \frac{1}{8}e^\Phi\left[F^{(0)} - \frac{1}{2!}\Gamma^{\mu\nu}F_{\mu\nu}^{(2)} + \frac{1}{4!}\Gamma^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}^{(4)}\right]\Gamma_\mu\epsilon, \quad (4.2)$$

where we have used μ, ν, ρ to describe the ten dimensional space-time indices, and the hatted ones are the corresponding tangent space indices. Solving the Dilatino variation (4.1) for the D4-brane solution (3.1), presented in [4] we get the following condition on the spinors to be satisfied

$$\begin{aligned} f^{-5/4}f_{,m}\left(\Gamma^{\hat{m}} + \frac{1}{4!}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}}\Gamma^{\hat{n}\hat{p}\hat{q}\hat{r}}\right)\epsilon - 2\mu f^{1/4}\left(\Gamma^{\hat{1}\hat{2}} + \Gamma^{\hat{3}\hat{4}}\right)\Gamma^{\hat{9}}\epsilon \\ - \mu f^{1/4}\left(\Gamma^{\hat{1}\hat{2}} + \Gamma^{\hat{3}\hat{4}}\right)\epsilon - \mu f^{-1/4}\Gamma^{\hat{0}}\left(\Gamma^{\hat{1}\hat{2}} + \Gamma^{\hat{3}\hat{4}}\right)\Gamma^{\hat{9}}\epsilon = 0 \end{aligned} \quad (4.3)$$

Now solving the gravitino variations we get the following

$$\partial_0\epsilon = 0, \quad \partial_a\epsilon = 0, \quad (a = 5, \dots, 9), \quad \partial_i\epsilon + \frac{\mu}{2}J_{ij}\Gamma^{\hat{j}\hat{9}}\epsilon = 0, \quad (i = 1, \dots, 4). \quad (4.4)$$

Note that while writing down the above variations (4.4) we have made use of the D4-brane supersymmetry condition in flat space

$$\left(\Gamma^{\hat{m}} + \frac{1}{4!}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}}\Gamma^{\hat{n}\hat{p}\hat{q}\hat{r}}\right)\epsilon = 0, \quad (4.5)$$

and

$$\left(1 - \Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\right)\epsilon = 0. \quad (4.6)$$

By using the above two conditions all the dilatino and gravitino variations are satisfied leaving only 1/4 of the total spacetime supersymmetry unbroken and is solved by a constant spinor. Hence the D4-brane solution in the $n = 2$ Gödel universe model preserves 1/4 unbroken supersymmetry. Let us now look at the fate of the unbroken supersymmetry for the D4-brane in $n = 1$ Gödel model. First, solving the dilatino variation (4.1), for the D4-brane solution presented in (3.3) we get

$$f^{-5/4}f_{,m}\left(\Gamma^{\hat{m}} + \frac{1}{4!}\epsilon_{mnpqr}\Gamma^{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}}\right)\epsilon - 2\mu f^{1/4}\Gamma^{\hat{1}\hat{2}}\left(\Gamma^{\hat{9}} + \frac{1}{2}(1 + \Gamma^{\hat{0}\hat{9}})\right)\epsilon = 0 \quad (4.7)$$

The vanishing of the dilatino variation demands that the following two conditions to be imposed

$$\left(\Gamma^{\hat{m}} + \frac{1}{4!}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}}\Gamma^{\hat{n}\hat{p}\hat{q}\hat{r}}\right)\epsilon = 0 \quad (4.8)$$

and

$$\Gamma^{\hat{0}}\epsilon = \Gamma^{\hat{9}}\epsilon = -\epsilon \quad (4.9)$$

The first one is the usual D4-brane supersymmetry condition even in flat space, where as the second condition is a projection condition on the spinors. By using (4.8) and (4.9), all the gravitino variations are satisfied leaving the following equations to have a constant spinor as a solution.

$$\partial_0\epsilon = 0, \quad \partial_\alpha\epsilon = 0, \quad (\alpha = 3, \dots, 9), \quad \partial_i\epsilon + \frac{\mu}{2}J_{ij}\Gamma^{\hat{j}\hat{9}}\epsilon = 0, \quad (i = 1, 2). \quad (4.10)$$

Hence the D4-brane in the $n = 1$ Gödel model preserves 1/4 of the total spacetime supersymmetry. Similarly one can analyze the spacetime supersymmetry of the D2-brane presented in (3.10) by solving the dilatino and gravitino variations.

5 Conclusions

We have presented in this paper a class of Gödel universe backgrounds from non-standard intersecting branes in supergravity. These supersymmetric backgrounds are different from the already known ones due to the presence of various constant NS-NS and R-R field strengths. We have further presented the supergravity solutions of D-branes in type IIA theory in some Gödel universes which are obtained from the corresponding PP-wave backgrounds. The supersymmetry properties of these branes are analyzed in detail by solving the dilatino and gravitino variations explicitly. The worldsheet construction of these branes can be carried out by following [6][4] and looking at the mixed boundary conditions properly. It will be interesting to completely classify all the supersymmetric branes in Gödel universes of various kind.

Acknowledgements: We would like to thank S. F. Hassan and Aalok Misra for various useful discussions. KLP would like to thank the Abdus Salam I.C.T.P, Trieste for hospitality under Associate Scheme, where a part of this work was completed.

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